Matched Z Transform →

In this analog filter is transformed into the digital filter by mapping poles and zeros of H(S) directly into the poles and zeros in Z plane. Consider that the transfer fn of analog filter is given by

$$H(S) = \frac{\sum_{K=1}^{M} (S - Z_K)}{\sum_{K=1}^{M} (S - P_K)} \quad \text{—(1)}$$

Here Z_K represents the zeros of system and P_K poles of system

In this method the poles obtained from matched Z transform are identical to poles obtained with impulse invariance method. But the zeros positions are different than impulse invariance method.

The transfer fn of digital filter can be given by

$$H(Z) = \frac{\sum_{K=1}^{M} (1 - e^{Z_K T_S} \cdot Z^{-1})}{\sum_{K=1}^{M} (1 - e^{P_K T_S} \cdot Z^{-1})}$$

Here Ts is the sampling time. In this method, to maintain the freq response characteristics of analog filter normal, the sampling interval (i.e. Ts) is kept very low in order to avoid aliasing.

Butterworth Filter →

Design eq'ns and design steps

A_P = Attenuation in pass band

A_S = " stop "

Ω_p = Pass band edge freq

Ω_c = Cut off freq

Ω_s = stop band edge freq

Step 1 From the given specifications of digital filter obtain equivalent analog filter

(a) for impulse invariance method

$$\Omega = \frac{\omega}{T_s}$$

(b) for BLT ⇒ $\Omega = \frac{2}{T_s} \tan \frac{\omega}{2}$

Ω = freq of analog filter

ω = freq of digital filter

T_s = sampling time

Step II Calculate the order N of filter using the equation

$$N = \frac{1}{2} \times \frac{\log \left[\dfrac{\left(\dfrac{1}{A_s^2} - 1\right)}{\left(\dfrac{1}{A_p^2} - 1\right)} \right]}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

or $N = \dfrac{\log\left(\left(1 - \delta_s^2\right) - 1\right]}{2\log\left(\Omega_s / \Omega_c\right)}$ Here δ_s = Attenuation in stop band

If the specifications are given in decibels (dB) then use the eq'n

$$N = \frac{1}{2} \frac{\log\left[\dfrac{10^{0.1 A_s(dB)} - 1}{10^{0.1 A_p(dB)} - 1}\right]}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

Step 3 Calculations of Cut off freq (Ω_c)

The cut-off freq Ω_c of analog filter is Calculated as:-

(a) for IIM $\Rightarrow$ $\Omega_c = \dfrac{w_c}{T_s}$

(b) for BLT

$\Omega_c = \dfrac{2}{T_s} \tan \dfrac{w_c}{2}$

When W_c is not given then use the eq'n

(i) $\Omega_c = \dfrac{\Omega_p}{\left(\dfrac{1}{A_p^2} - 1\right)^{\frac{1}{2N}}}$ and if specifications are in dB then

(ii) $$\Omega_c = \frac{\Omega_p}{10^{0.1A_p} - 1}$$

Step 4 Calculate the poles using

$$P_K = \pm \Omega_c e^{j(N+2K+1)\pi/2N} \qquad K = 0, 1, 2, \dots N-1$$

If the poles are complex conjugate then organize the poles(P_K) as complex conjugate pairs that means

S_1 and S_1^* $\qquad$ S_2 and S_2^*

Step 5 Calculate the system transfer fn of analog filter using

$$H(S) = \frac{\Omega_c^N}{(S-P_1)(S-P_2)}$$

and if poles are complex conjugate then

$$H(S) = \frac{\Omega_c^N}{(S-S_1)(S-S_1^*)(S-S_2)(S-S_2^*)}$$

Step 6 design the digital filter using IIM or BLT

A digital filter has freq specification as
Passband freq = $\omega_p = 0.2\pi$
Stopband freq = $\omega_s = 0.3\pi$
What are the corresponding specification
1. IIM is used for designing
2. BLT is used for designing

Soln Assume sampling $T_s = 1$

For IIM $\Omega_p = \frac{\omega_p}{T_s}$ $\therefore \Omega_p = 0.2\pi$
$= 0.63$ rad/sec

$\Omega_s = \frac{W_s}{T_s} = \Omega_s = \frac{0.3\pi}{T_s} = 0.3\pi$

$\Omega_s = 0.94$ rad/sec

For BLT $\Omega_p = \frac{2}{T_s} \tan\left(\frac{\omega_p}{2}\right)$

$\Omega_p = 2\tan\left(\frac{0.2\pi}{2}\right) = 2\tan\left(\frac{0.2 \times 180}{2}\right)$

$\Omega_p = 0.65$ rad/sec

$\Omega_s = \frac{2}{T_s} \tan\left(\frac{W_s}{2}\right)$

$W_s = 2\tan\left(\frac{0.3\pi}{2}\right) = 2\tan\left(\frac{0.3 \times 180}{2}\right)$

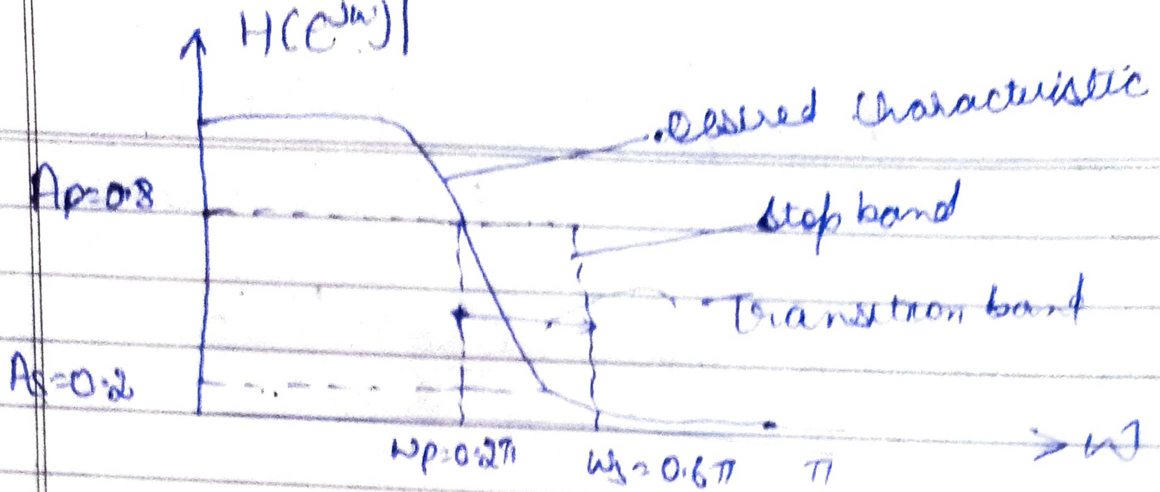
$\Omega_s = 1.019$ rad/sec

Using billinear transformation design a butterworth filter which satisfies the following conditions

$0.8 \leq |H(e^{jw})| \leq 1$ $\quad 0 \leq w \leq 0.2\pi$

$|H(e^{jw})| \leq 0.2$ $\quad 0.6\pi \leq w \leq \pi$

Soln: $H(e^{j\omega})$ is same as $|H(\omega)|$; so.



Given specifications of digital filter are

$A_p = 0.8, \quad \omega_p = 0.2\pi, \quad A_s = 0.2, \quad \omega_s = 0.6\pi$

Part A Step 1 Calculating specifications of analog filter

We will obtain the values of Ω_p and Ω_s. We have to use BLT

$\Omega = \frac{2}{T_s} \tan \frac{\omega}{2}$ $\Omega_p = \frac{2}{T_s} \tan \frac{\omega_p}{2}$

The value of sampling time Ts is not given so assume $T_s = 1$

$\Omega_p = 2 \tan \frac{0.2\pi}{2}$ $\therefore \Omega_p = 0.65$

Similarly $\Omega_s = \frac{2}{T_s} \tan \frac{\omega_s}{2} = 2 \tan \frac{0.6\pi}{2}$

$\Omega_s = 2.75$

Thus specifications of analog filter are

$A_p = 0.8$ $\Omega_p = 0.65$ $A_s = 0.2$ $\Omega_s = 2.75$

Step 2 Calculation of order of filter N

It is given by

$$N = \frac{1}{2} \times \frac{\log\left[\left(\frac{1}{A_s^2} - 1\right) / \left(\frac{1}{A_p^2} - 1\right)\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N = \frac{1}{2} \times \frac{\log\left[\left(\frac{1}{(0.2)^2} - 1\right) / \left(\frac{1}{(0.8)^2} - 1\right)\right]}{\log\left(\frac{2.75}{0.65}\right)}$$

$$= \frac{1}{2} \times \frac{\log\left[\frac{24}{0.5625}\right]}{\log\left(\frac{2.75}{0.65}\right)} = 1.3$$

approximately we can take $N = 2$

Step 3 Calculating of cut-off freq Ω_c

W_c is not given so use the eq'n

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_p^2} - 1\right)^{1/2N}} = \frac{0.65}{\left[\frac{1}{(0.8)^2} - 1\right]^{\frac{1}{4}}} = 0.75$$

Step 4 Calculations of the poles of $H(s)$

$P_K = \pm \Omega_c \, e^{j(N+2K+1)\pi/2\pi}$ $\quad K = 0, 1, \ldots, N-1$

$P_K = \pm 0.75 \, e^{j(2+2K+1)\pi/4}$

$P_K = \pm 0.75 \, e^{j(3+2K)\pi/4}$

Here range of k is from 0 to $N-1$ and $N=2$. Thus the value of k is $k=0$ and $k=1$

For $k=0$

$P_0 = \pm 0.75 e^{j(3)\pi/4}$

$= \pm 0.75\left(\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right]$

$P_0 = \pm 0.75[-0.707 + j0.707]$

$P_0 = \pm [-0.53 + j0.53]$

$P_0 = -0.53 + j0.53$ and $0.53 - j0.53$

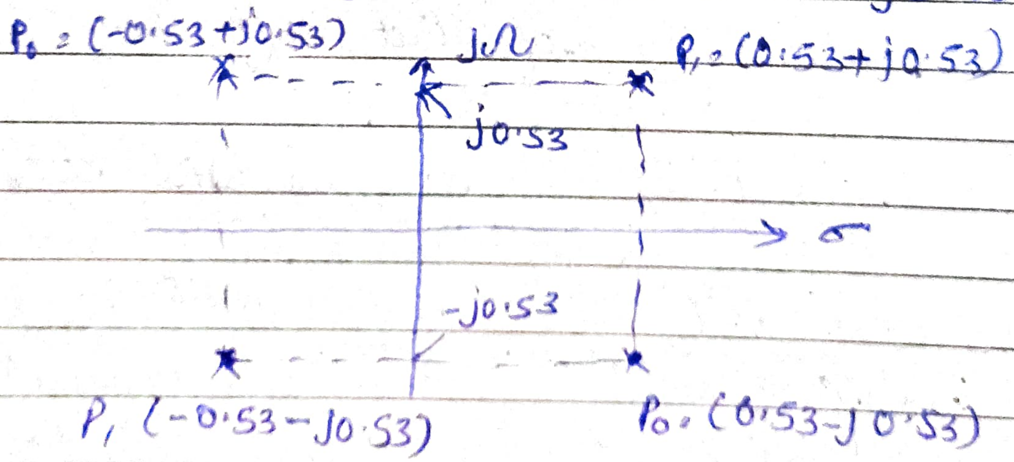
For $k=1$

$P_1 = \pm 0.75 e^{j(3+2)\pi/4} = \pm 0.75 e^{j5\pi/4}$

$P_1 = \pm 0.75\left[\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4}\right]$

$P_1 = \pm 0.75[-0.707 - j0.707]$

$P_1 = -0.53 - j0.53$ and $0.53 + j0.53$



$S_1 = -0.53 + j0.53$

$S_1^* = -0.53 - j0.53$

Step 5 Calculation of $H(S)$

The transfer fn of analog filter is given by

$$H(S) = \frac{\Omega_c^N}{(S-S_1)(S-S_1^*)}$$

$$H(S) = \frac{(0.75)^2}{(S+0.53-j0.53)(S+0.53+j0.53)}$$

$$H(S) = \frac{(0.75)^2}{S^2+0.53S+j0.53S+0.53S+0.28+j0.28-j0.53S-j0.28+0.28}$$

$$H(S) = \frac{(0.75)^2}{S^2+1.06S+0.56}$$

$$H(S) = \frac{0.56}{S^2+1.06S+0.56}$$

Part B We have to use BLT. Then $H(Z)$ is obtained by putting $S = \frac{2}{Ts}\left(\frac{Z-1}{Z+1}\right)$ in the eq'n $H(S)$

$$H(Z) = \frac{0.56}{4\left(\frac{Z-1}{Z+1}\right)^2 + (2\times1.06)\left(\frac{Z-1}{Z+1}\right) + 0.56}$$

$$H(Z) = \frac{0.56}{\frac{4(Z-1)^2}{(Z+1)^2} + \frac{2.12Z-2.12}{Z+1} + 0.56}$$

$$H(Z) = \frac{0.56(Z+1)^2}{4(Z^2 - 2Z + 1) + (2.12Z - 2.212)(Z+1) + 0.56(Z+1)^2}$$

$$H(z) = \frac{0.56(Z+1)^2}{4z^2 - 8z + 4 + 2.12z^2 + 2.12z - 2.12z - 2.12 + 0.56z^2 + 1.12z + 0.56}$$

$$H(Z) = \frac{0.56(Z+1)^2}{6.68z^2 - 6.88z + 2.44}$$

Analog and digital Transformation in freq Domain $\Rightarrow$

Freq transformation in Analog Domain

we have normalized LPF having cut-off freq Ω_c

Low pass to low Pass $\Rightarrow$

$$s = \frac{\Omega_c}{\Omega_{LP}} s$$

Low Pass to high Pass $\Rightarrow$

HPF with cutoff freq Ω_{HP}

$$s = \frac{\Omega_c \Omega_{HP}}{s}$$

3. Low Pass to band Pass

$$S \Rightarrow \Omega_c \frac{S^2 + \Omega_l \Omega_u}{S(\Omega_u - \Omega_l)}$$

4 Lowpass to bandstop or notch or band reject

$$S \Rightarrow \Omega_c \frac{S(\Omega_u - \Omega_l)}{S^2 + \Omega_u \Omega_l}$$

Frequency Transformation in Digital Domain →

1. Low Pass to low Pass →

$$Z^{-1} \Rightarrow \frac{Z^{-1} \alpha}{1 - \alpha Z^{-1}}$$

$$\text{Here } \alpha = \frac{\sin\left(\frac{w_p' - w_p}{2}\right)}{\sin\left(\frac{w_p' + w_p}{2}\right)}$$

w_p' = Pass band freq of LPF

w_p = " " " new filter

2. Low pass to high Pass

$$Z^{-1} \rightarrow \frac{-Z^{-1} + \alpha}{1 + \alpha Z^{-1}}$$

$$\text{Here } \alpha = \frac{-\cos\left(\frac{w_p' + w_p}{2}\right)}{\cos\left(\frac{w_p' + w_p}{2}\right)}$$

w_p' → Pass band freq of low Pass filter

w_p = " " " " High "

3 Low pass to band pass →

$$Z^{-1} \to \frac{-Z^{-2} - \frac{2\alpha K}{K+1} Z^{-1} + \frac{K-1}{K+1}}{\frac{K-1}{K+1} Z^{-2} - \frac{2\alpha K}{K+1} Z^{-1} + 1}$$

$$\alpha = \frac{\cos\left(\frac{\omega_2 + \omega_1}{2}\right)}{\cos\left(\frac{\omega_2 - \omega_1}{2}\right)}$$

And $K = \cot\left(\frac{\omega_2 - \omega_1}{2}\right) \tan \frac{\omega_p'}{2}$

4 Low Pass to band reject →

$$Z^{-1} \to \frac{Z^{2} - \frac{2\alpha}{K+1} Z^{-1} + \frac{1-K}{1+K}}{\frac{1-K}{1+K} Z^{-2} - \frac{2\alpha}{1+K} Z^{-1} + 1}$$

Here $\alpha = \frac{\cos\left(\frac{\omega_2 + \omega_1}{2}\right)}{\cos\left(\frac{\omega_2 - \omega_1}{2}\right)}$

$$K = \tan\left(\frac{\omega_2 - \omega_1}{2}\right) \tan \frac{\omega_p'}{2}$$

Design the digital high pass filter for cut off freq of 30HZ and sampling freq of 150HZ using BLT

Soln **Part A** Calculations of specifications for digital filter

$$f_{HP} = \frac{F_{HP}}{F_s} \qquad F_{HP} = 30\,Hz \quad F_s = 150\,Hz$$

$$f_{HP} = \frac{30}{150} = 0.2 \text{ cycles/ Samples}$$

$$\omega_{HP} = 2\pi f_{HP} = 2\pi \times 0.2 = 0.4\pi$$

The order of filter is not given. so we can assume order of $N=1$

Part B Calculation of specifications for analog filter

Step 1 Calculation of Ω_{HP}

$$\Omega = \frac{2}{T_s} \tan\frac{\omega}{2}$$

$$\Omega_{HP} = \frac{2}{\frac{1}{150}} \tan\frac{\omega_{HP}}{2} = 300 \tan\frac{0.4\pi}{2}$$

$$\Omega_{HP} = 217.96 \text{ rad/sec}$$

we assumed $N=1$

In of prototype L.P.F

$$H(s) = \frac{1}{s-s_1} = \frac{1}{s+1}$$

Part C use freq transformation

$$s \rightarrow \frac{\Omega_c \Omega_{HP}}{s}$$

$$\Omega_c = 1$$

$$s = \frac{\Omega_{HP}}{s} \Rightarrow s = \frac{217.96}{s}$$

Putting this value in eqⁿ ① we get

$$H(S) = \frac{1}{\frac{217.96}{s}+1} = \frac{S}{S+217.96} \quad \text{—} (2)$$

Part D Calculation of H(Z) for digital filter using BLT

$$S = \frac{2}{Ts}\left[\frac{Z-1}{Z+1}\right] \quad \text{in eq}^n \text{ (2)}$$

$$H(Z) = \frac{2\times150\left(\frac{Z-1}{Z+1}\right)}{2\times150\left(\frac{Z-1}{Z+1}\right)+217.96} = \frac{300\left(\frac{Z-1}{Z+1}\right)}{300\left(\frac{Z-1}{Z+1}\right)+217.96}$$

$$H(Z) = \frac{\left(\frac{Z-1}{Z+1}\right)}{\left(\frac{Z-1}{Z+1}\right)+0.73}$$

$$H(Z) = \frac{Z-1}{\{Z-1\}+0.73Z+0.73}$$

$$H(Z) = \frac{Z-1}{1.73Z-0.27}$$

Comparision b/w IIR and FIR filter

Char	IIR	FIR
1. Number of necessary multiplications	least	Most
2. Sensitivity of filter coefficient quantization	Can be high for direct form	very low
3. Probability of over flow errors	Can be high for direct form	very low
4. Stability	Depended upon system design	Guaranted
5. Linear Phase	No	Guranted
6. Can simulate prototype Analog filter	Yes	No
7. Required memory	Least	Most
8. H/w filter control complexity	Moderate	Simple
9. Availability of design software	Good	very good